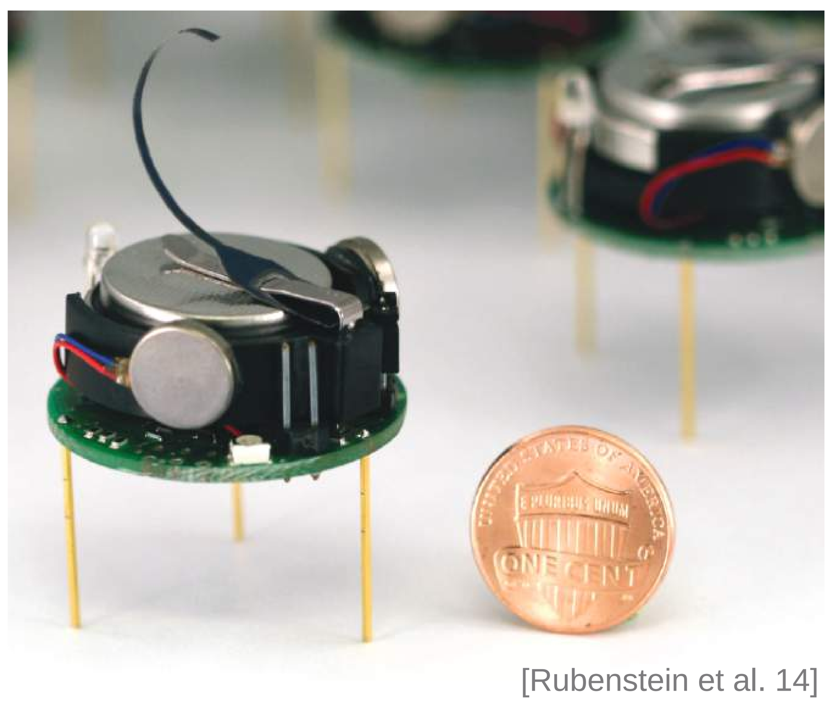
Focal Flow

Measuring Depth and Velocity with Defocus and Differential Motion

Emma Alexander[1], Qi Guo[1], Sanjeev Koppal[2], Steven Gortler[1], Todd Zickler[1]

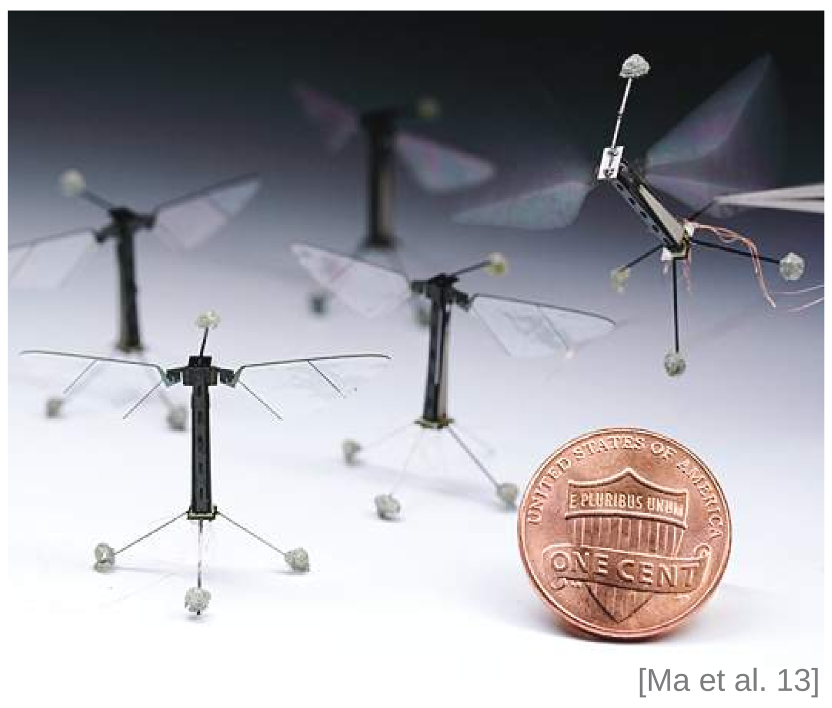
1 Harvard SEAS
2 University of Florida

Motivation — Low power (mW) depth sensing



[Rubenstein et al. 14]

~200 mW



[Ma et al. 13]

~20 mW

Contribution

Optical Flow — Focal Flow

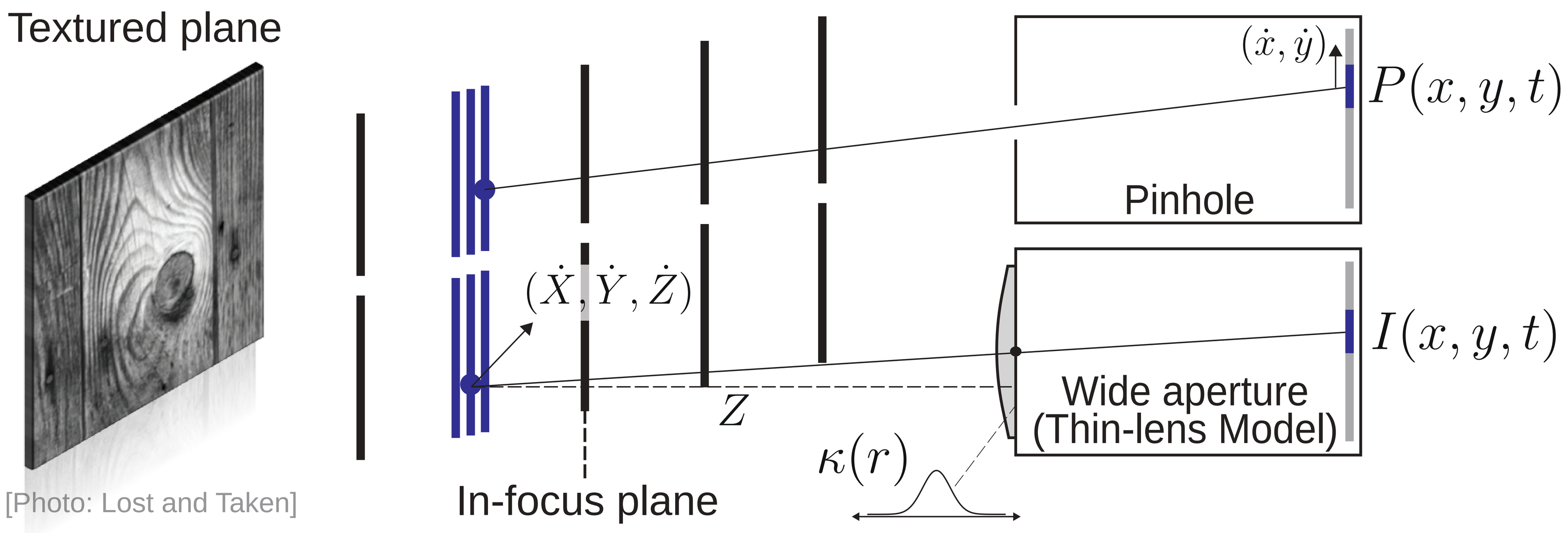
$$0 = \left[\boxed{I_x \quad I_y} \quad (xI_x + yI_y) \quad (I_{xx} + I_{yy}) \right] \vec{u}_{4\times1} + I_t$$

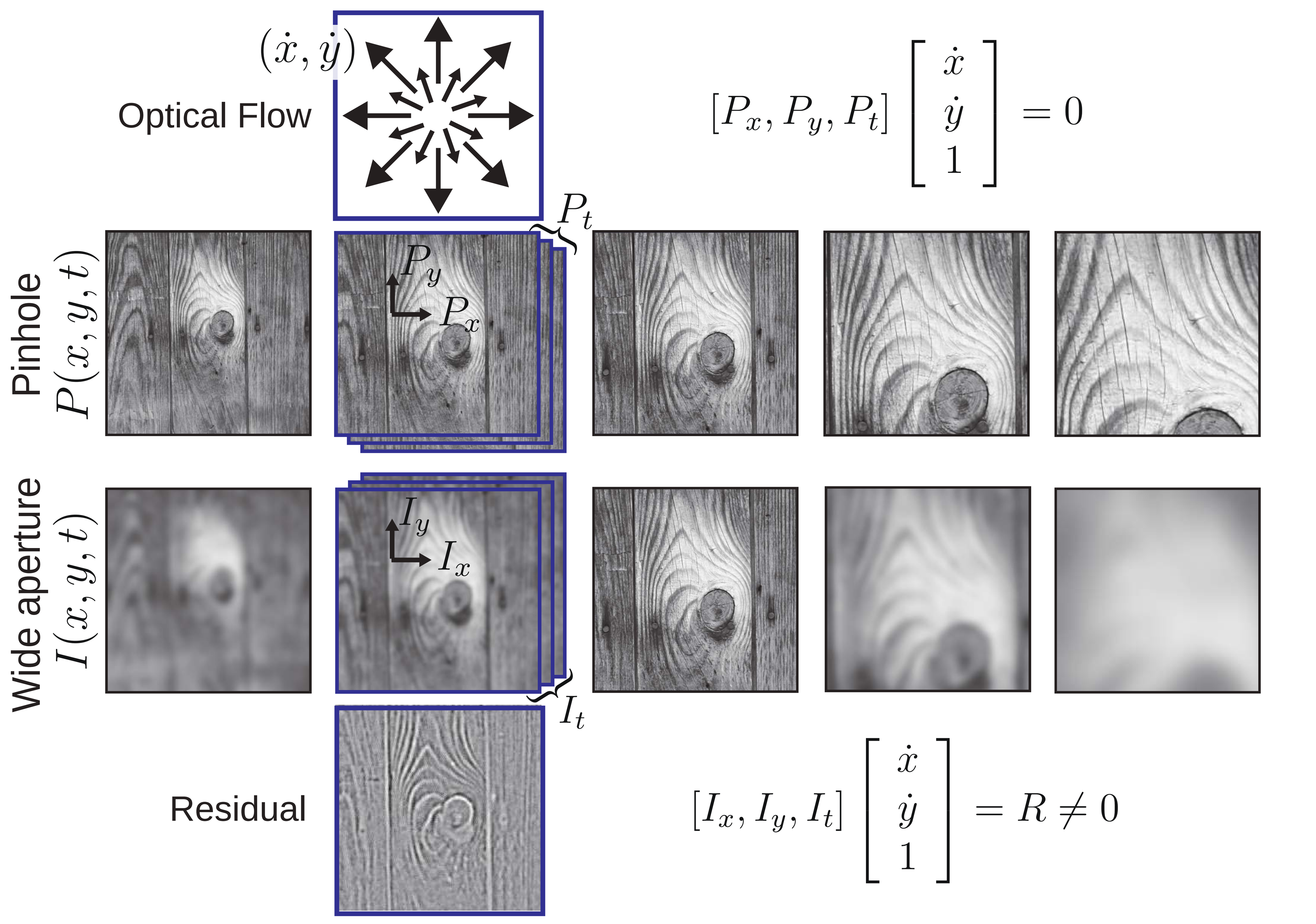
Image Motion — Depth & 3D Velocity

[Photo: Tony Hisgett]

$$A_{4\times4}\vec{u}_{4\times1} = \vec{b}_{4\times1}$$
$$\downarrow$$
$$Z, (\dot{X}, \dot{Y}, \dot{Z})$$

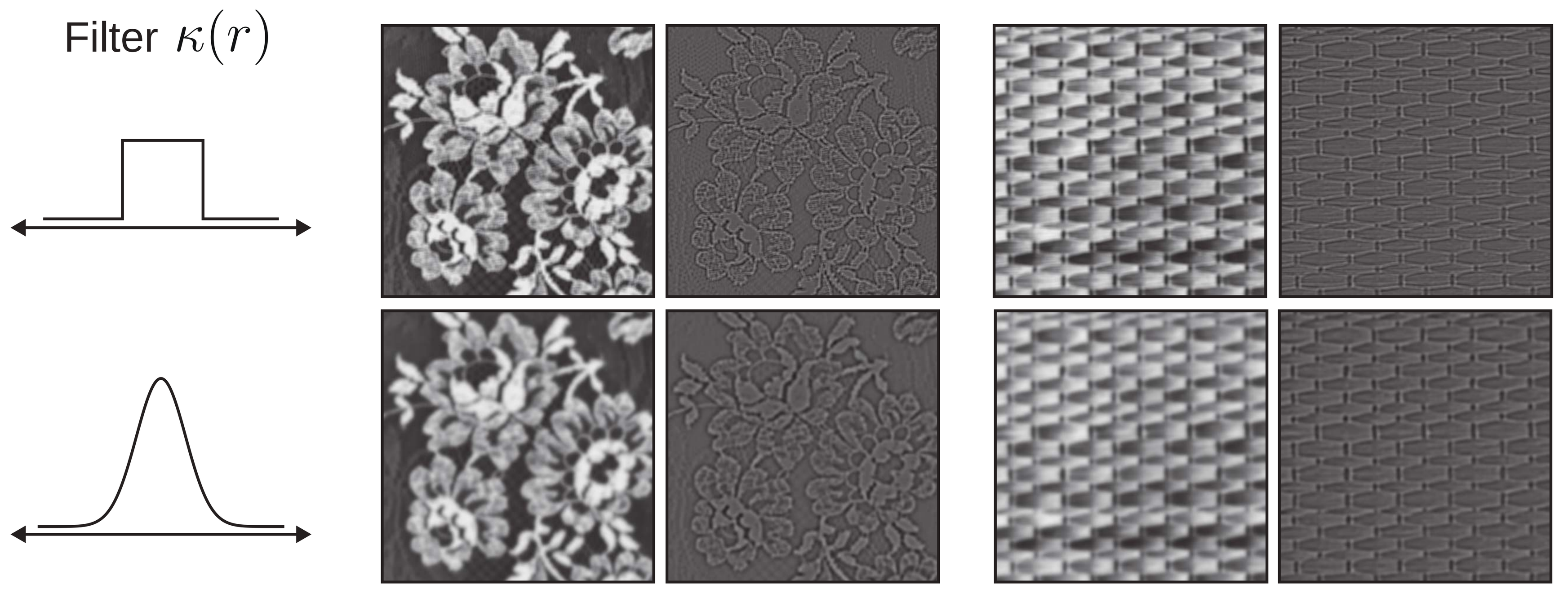
Idea — Combine motion and defocus blur





Derivation — Gaussian blur reveals depth

Filter $\kappa(r)$



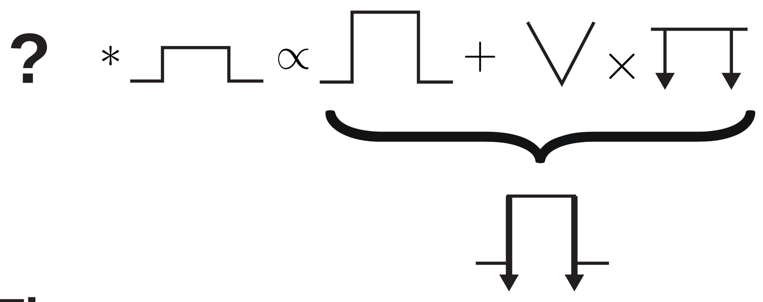
$$I_t + xI_x + yI_y = R(x,y,t;P,\kappa,Z,\dot{Z}) = \frac{\dot{Z}}{Z-\mu_f}\frac{1}{\sigma^2(Z)}\left(2\kappa\left(\frac{r}{\sigma(Z)}\right) + \frac{r}{\sigma(Z)}\kappa'\left(\frac{r}{\sigma(Z)}\right)\right) * P$$

$$R(Z, \dot{\vec{X}}, \text{filter } \kappa, \text{pinhole image } P) \propto m * I$$

$$= \frac{\dot{Z}}{Z - Z_f}(2k + rk_r) * P \qquad = v(Z, \dot{\vec{X}})\; m * k * P$$

$m * k \propto 2k + rk_r$	Texture independence
$w\hat{m}\hat{k} = -\hat{r}\hat{k}_{\hat{r}}$	Fourier transform
$\hat{k} \propto e^{-w\int_0^{\hat{r}} \frac{\hat{m}(s)}{s} ds}$	Solve differential equation
$\hat{m} \propto \hat{r}^n$	All kernels k from same filter κ
$n \in \{2, 4, 6, ...\}$	Compact operator
$n = 2$	Nonnegative transmittance
$\hat{m} = \hat{r}^2 \rightarrow m = \nabla^2$	Inverse Fourier Transform

$$m \;*\; k \;\propto\; 2k \;+\; r \;\; k_r$$



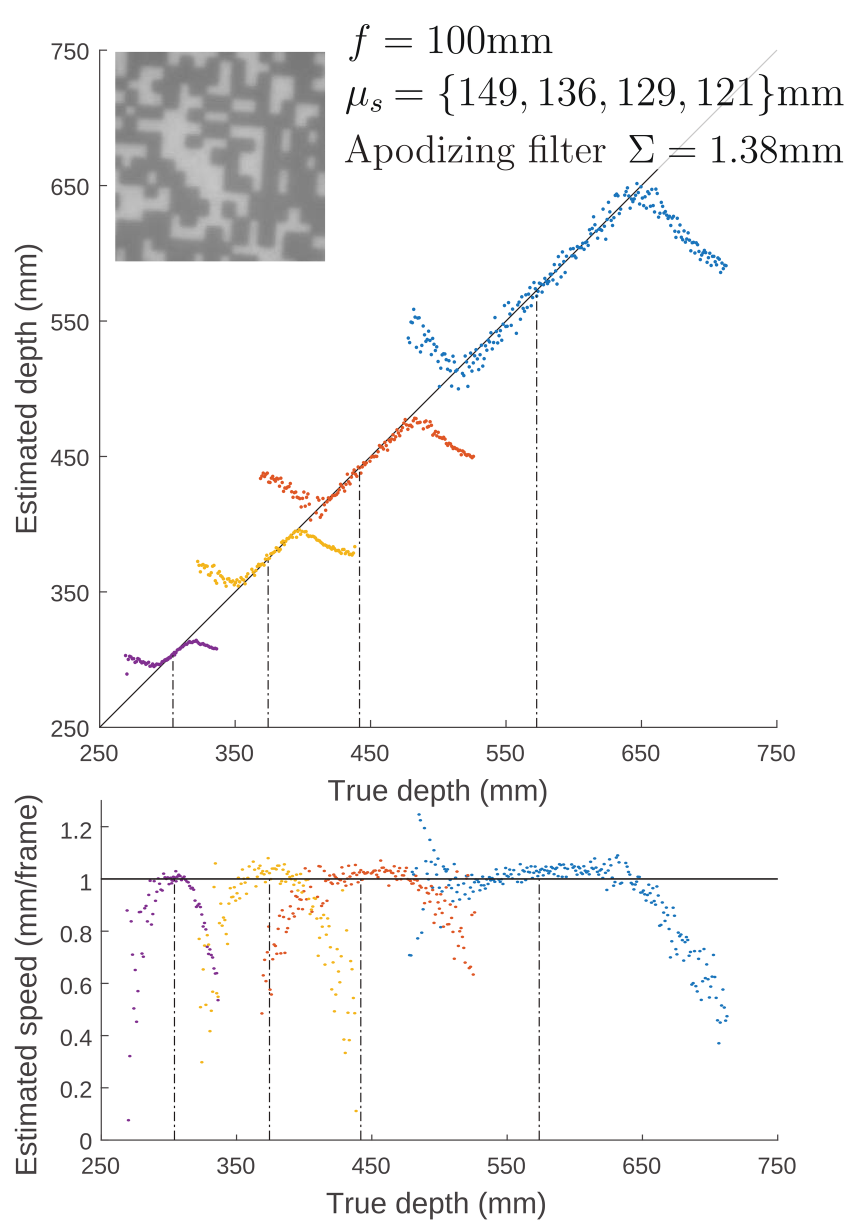
Theorem

The residual can be factored into scene information and an image convolution exactly when the blur is **Gaussian** and the operator is the **Laplacian**, i.e.

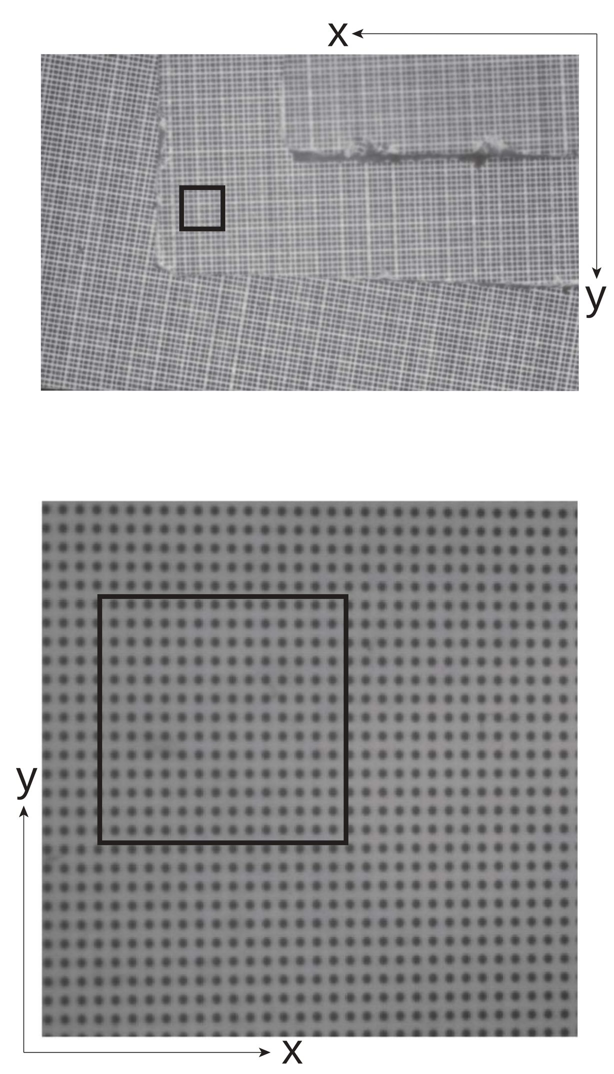
$$\kappa = \exp\left(-\Sigma^2 r^2\right),$$
$$m = \nabla^2.$$

$$0 = [I_x \quad I_y \quad (xI_x + yI_y) \quad (I_{xx} + I_{yy})]\,\vec{u}_{4\times1} + I_t\,, \quad \vec{u}_{4\times1} = -\left[\frac{\dot{X}\mu_s}{Z}, \frac{\dot{Y}\mu_s}{Z}, \frac{\dot{Z}}{Z}, \frac{\dot{Z}}{Z}\left(1 - \frac{\mu_f}{Z}\right)\left(\frac{\Sigma\mu_s}{\mu_f}\right)^2\right]^T.$$

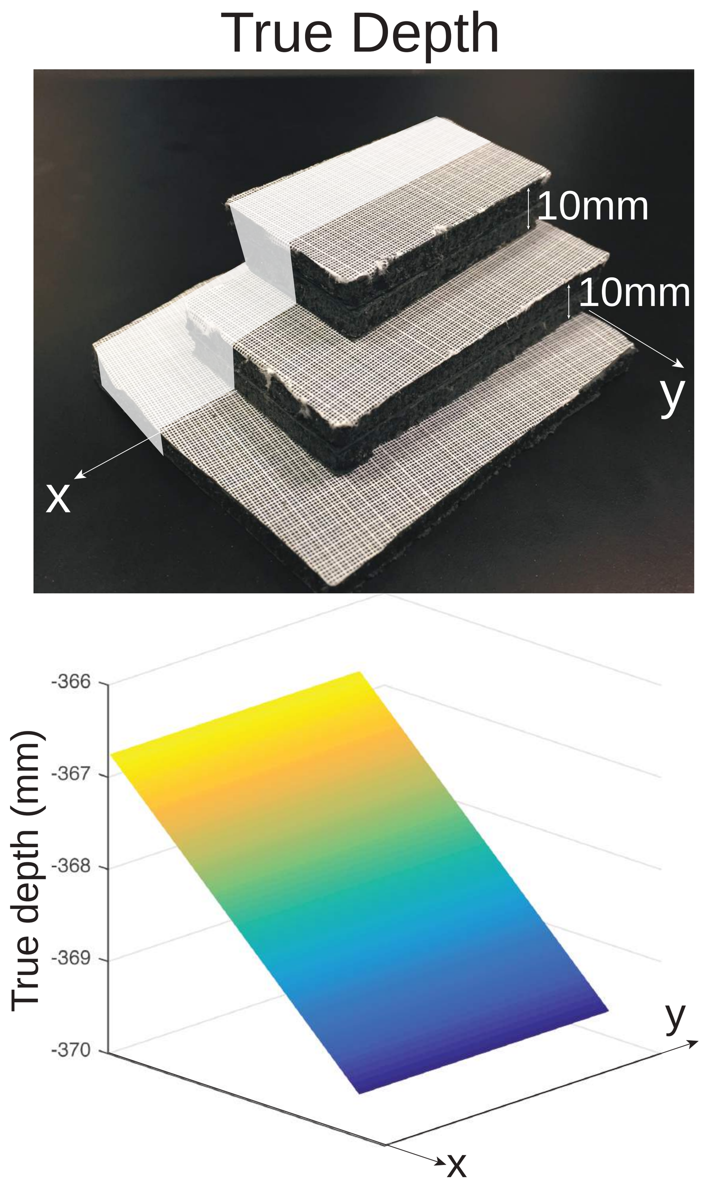
Proof of Concept — Experimental results



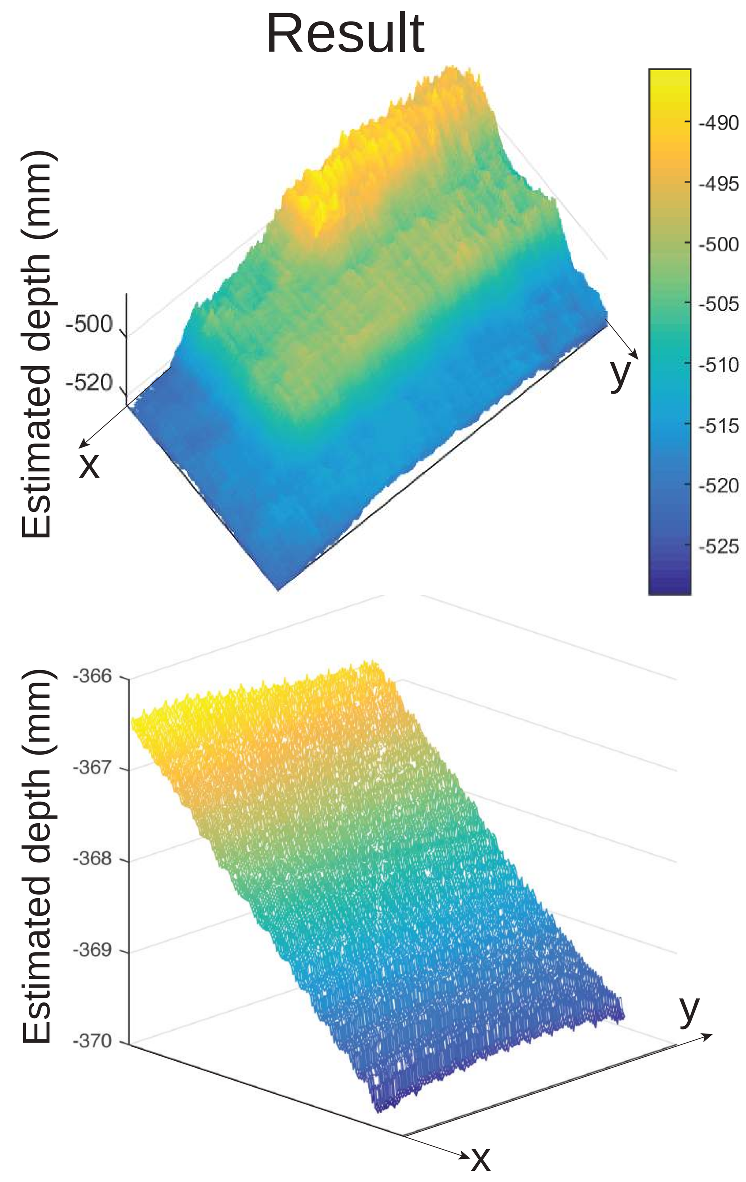
Input Image



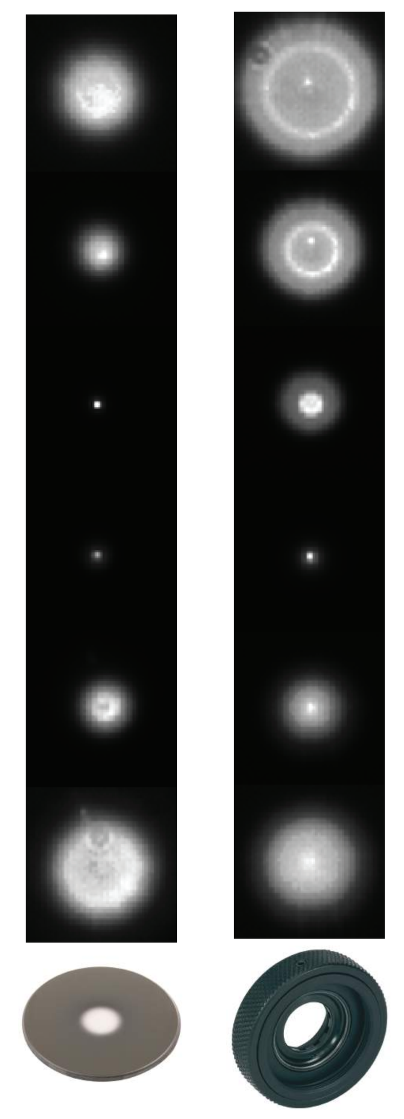
True Depth



Result



Sample PSF



[Photo: Thorlabs]

Code, equipment, results: https://vision.seas.harvard.edu/focalflow